JEE/BITSAT LEVEL TEST

Booklet Code A/B/C/D **Test Code : 2003**

Matrices & Determinants

Answer Key/Hints

Q.1 If A = $\begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}$, then A' A is equal to

(a.) I (b.) -iA

(c.) -I (d.) iA

Sol. We have $A'A = \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I$

Q.2 Let $E(\alpha) = \begin{bmatrix} \cos^2\alpha & \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha \end{bmatrix}$. If α and β differs by an odd multiple of $\pi/2$, then $E(\alpha)\,E(\beta)$ is a

(a.) null matrix (b.) unit matrix (c.) Diagonal Matrix (d.) Orthogonal Matrix

Sol. We have $E(\alpha)E(\beta)$

$$= \begin{bmatrix} \cos^2\alpha & \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha \end{bmatrix}\begin{bmatrix} \cos^2\beta & \cos\beta\sin\beta \\ \cos\beta\sin\beta & \sin^2\beta \end{bmatrix} = \begin{bmatrix} \cos\alpha\cos\beta\cos(\alpha-\beta) & \cos\alpha\sin\beta\cos(\alpha-\beta) \\ \sin\alpha\cos\beta\cos(\alpha-\beta) & \sin\alpha\sin\beta\cos(\alpha-\beta) \end{bmatrix}$$

As α and β differ by an odd multiple of $\pi/2$, $\alpha-\beta = (2n+1)\pi/2$ for some integer n.

Thus, cos $[(2n+1)\pi/2] = 0$ $\therefore$ $E(\alpha)E(\beta) = \boldsymbol{0}$

Q.3 If $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$, then the value of $\frac{x^2+y^2}{xy}$ is equal to:

(a.) $\frac{5}{2}$ (b.) $-\frac{3}{2}$ (c.) $\frac{13}{6}$ (d.) $-\frac{13}{6}$

Sol. $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix} \Rightarrow \begin{bmatrix} x+2y \\ 2x+y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix} \Rightarrow x+2y=5,\ 2x+y=4 \Rightarrow x=1, y=2 \Rightarrow \frac{x^2+y^2}{xy} = \frac{1+4}{2.1} = \frac{5}{2}$

Q.4 If for the matrices A and B, AB = A and BA = B, then A^2 is equal to

(a.) I (b.) A (c.) B (d.) none of these

Sol. AB = A, BA = BA^2 = AA = (AB)A = A (BA) = AB = A

Q.5 The value of x for which the matrix $\begin{bmatrix} 2/x & -1 & 2 \\ x & x & 2x^2 \\ 1 & 1/x & 2 \end{bmatrix}$ is singular is

(a.) ± 1 (b.) ± 2 (c.) ± 3 (d.) None of these

Sol. . As $|A| = \left(\frac{2}{x}\right)\begin{vmatrix} x & 2x^2 \\ 1/x & 2 \end{vmatrix} + \begin{vmatrix} 1 & 2x^2 \\ 1 & 2 \end{vmatrix} + 2\begin{vmatrix} 1 & x \\ 1 & 1-x \end{vmatrix} = \frac{2}{x}(0) + 2 - 2x^2 + 2\left(\frac{1}{x} - x\right)$

$= \frac{2x(1-x^2) + 2(1-x^2)}{x} = \frac{2(1+x)^2(1-x)}{x}$ Now, $|A| = 0 \Rightarrow x = \pm 1$.

Q.6 If $A.\begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 7 & 7 \end{bmatrix}$, then A is equal to:

(a.) $\begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$ (b.) $\begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}$ (c.) $\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ (d.) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

Sol. Let $B = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} \Rightarrow adj(B) = \begin{bmatrix} 1 & -3 \\ -2 & -1 \end{bmatrix}^T = \begin{bmatrix} 1 & -2 \\ -3 & -1 \end{bmatrix}$

Also, |B| = –1 – 6 = –7 $\Rightarrow$ $B^{-1} = -\frac{1}{7}\begin{bmatrix} 1 & -2 \\ -3 & -1 \end{bmatrix}$ Also, |B| = – 1 – 6 = – 7

$\Rightarrow$ $B^{-1} = -\frac{1}{7}\begin{bmatrix} 1 & -2 \\ -3 & -1 \end{bmatrix}$ We have, $AB = \begin{bmatrix} -4 & 1 \\ 7 & 7 \end{bmatrix} \Rightarrow A = \begin{bmatrix} -4 & 1 \\ 7 & 7 \end{bmatrix}B^{-1} = -\frac{1}{7}\begin{bmatrix} -4 & 1 \\ 7 & 7 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -3 & -1 \end{bmatrix}$

$= -\frac{1}{7}\begin{bmatrix} (-4).1 + 1.(-3) & (-4).(-2) + 1.(-1) \\ 7.1 + 7.(-3) & 7.(-2) + 7.(-1) \end{bmatrix} = -\frac{1}{7}\begin{bmatrix} -7 & 7 \\ -14 & 21 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

Q.7 If $A(x_1) = \begin{bmatrix} \cos x_1 & -\sin x_1 & 0 \\ \sin x_1 & \cos x_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $A(x_2) = \begin{bmatrix} \cos x_2 & 0 & \sin x_2 \\ 0 & 1 & 0 \\ -\sin x_2 & 0 & \cos x_2 \end{bmatrix}$, then $(A(x_1) . A(x_2))^{-1}$

(a.) $A(x_1) . A(x_2)$ (b.) $A(-x_1) . A(-x_2)$

(c.) $- A(x_2) . A(x_1)$ (d.) $A(-x_2) . A(-x_1)$

Sol. $A(x_1)| = \cos^2 x_1 + \sin^2 x_1 = 1$

adj $(A(x_1)) = \begin{bmatrix} \cos x_1 & -\sin x_1 & 0 \\ \sin x_1 & \cos x_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} \cos x_1 & \sin x_1 & 0 \\ -\sin x_1 & \cos x_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow A^{-1}(x_1) = \frac{1}{|A(x_1)|} adj(A(x_1))$

$= \begin{bmatrix} \cos x_1 & \sin x_1 & 0 \\ -\sin x_1 & \cos x_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A(-x_1)$ Now, $|A(x_2)| = \cos^2 x_2 + \sin^2 x_2 = 1$ $adj(A(x_2)) = \begin{bmatrix} \cos x_2 & 0 & \sin x_2 \\ 0 & 1 & 0 \\ -\sin x_2 & 0 & \cos x_2 \end{bmatrix}^T$

$= \begin{bmatrix} \cos x_2 & 0 & -\sin x_2 \\ 0 & 1 & 0 \\ \sin x_2 & 0 & \cos x_2 \end{bmatrix} = A(-x_2)$ Finally, $(A(x_1) . A(x_2))^{-1} = A^{-1}(x_2) . A^{-1}(x_1) = A(-x_2) . A(-x_1)$

Q.8 The number of distinct values of a 2×2 determinant whose entries are from the set {-1, 0, 1} is:

(a.) 3 (b.) 4 (c.) 5 (d.) 6

Sol.

Possible values are -2, -1, 0, 1, 2 i.e. $\begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix} = 1, \begin{vmatrix} 1 & -1 \\ 0 & 0 \end{vmatrix} = 0, \begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} = -1 \quad \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2, \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -2$

Q.9 Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$. If $|A^2| = 25$ then $|\alpha| =$

(a.) 5^2 (b.) 1 (c.) 1/5 (d.) 5

Sol. $|A^2| = 25 \Rightarrow |A^2| = 25 \quad \therefore \quad (25\alpha)^2 = 25 \Rightarrow 25\alpha^2 = 1 \Rightarrow |\alpha| = 1/5$

Q.10 If C is a 3 x 3 matrix satisfying the relation $C^2 + C = I$, then C^{-2} is given by

(a.) 2C (b.) 3C-1 (c.) C (d.) 2I+C

Sol. $C^2 + C = I \Rightarrow C(C+I) = I \Rightarrow C^{-1} = C + I$ Thus, $C^{-2} = (C+I)^2 = C^2 - 2C + I$ = I - C + 2C + I = 2I + C

Q.11 If A, B and C are three square matrices of the same size such that B=CA C^{-1}, then $CA^3\,C^{-1}$ is equal to

(a.) B (b.) B^2 (c.) B^3 (d.) B^9

Sol. $CA^3C^{-1} = (CAC^{-1})(CAC^{-1})(CAC^{-1}) = B^3$

Q.12 The Values of a for which the matrix $A = \begin{pmatrix} a & a^2-1 & -3 \\ a+1 & 2 & a^2+4 \\ -3 & 4a & -1 \end{pmatrix}$ is symmetric

(a.) -1 (b.) -2 (c.) 3 (d.) 2

Sol. $A' = A$

$$\Rightarrow \begin{pmatrix} a & a+1 & -3 \\ a^2-1 & 2 & 4a \\ -3 & a^2+4 & -1 \end{pmatrix} = \begin{pmatrix} a & a^2-1 & -3 \\ a+1 & 2 & a^2+4 \\ -3 & 4a & -1 \end{pmatrix} \Rightarrow \quad a+1 = a^2-1 \quad \text{and} \quad 4a = a^2+4$$

$$\Rightarrow \quad a+1 = 4a-4-1 \quad \Rightarrow \quad 6-3a \Rightarrow a = 2.$$

Q.13 If the system of equations ax+y=3, x+2y=3, 3x+4y=7 is consistent, then value of a is given by

(a.) 2 (b.) 1 (c.) -1 (d.) 0

Sol. Solving x+2y=3, 3x+4y=7, we get x=1, y=1 $\therefore$ $a+1=3 \Rightarrow$ a=2.

Q.14 If $\begin{pmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{pmatrix}\begin{pmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{pmatrix}^{-1} = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$, then

(a.) a = b = 1
(b.) $a = \cos 2\theta,\ b = \sin 2\theta$
(c.) $a = \sin 2\theta, b = \cos 2\theta$
(d.) $a = 1, b = \sin 2\theta$

Q.15 Let M be a 3 x 3 matrix satisfying $M\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, M\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, and $M\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$ then sum of the diagonal entries of M is

(a.) 0
(b.) -3
(c.) 6
(d.) 9

Q.16 If $A = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$, then A^{-3} is

(a.) $\frac{1}{27}\begin{bmatrix} 1 & -26 \\ 0 & -27 \end{bmatrix}$
(b.) $\frac{1}{27}\begin{bmatrix} -1 & -26 \\ 0 & -27 \end{bmatrix}$
(c.) $\frac{1}{27}\begin{bmatrix} 1 & -26 \\ 0 & 27 \end{bmatrix}$
(d.) $\frac{1}{27}\begin{bmatrix} -1 & 26 \\ 0 & -27 \end{bmatrix}$

Sol. We have $A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$

$$\Rightarrow A^{-3} = \frac{1}{27}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} = \frac{1}{27}\begin{bmatrix} 1 & -8 \\ 0 & 9 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} = \frac{1}{27}\begin{bmatrix} 1 & -26 \\ 0 & 27 \end{bmatrix}$$

Q.17 The matrix A = $\begin{pmatrix} p & -q \\ q & p \end{pmatrix}$ is orthogonal if and only if

(a.) $P^2 + q^2 = 1$
(b.) $P^2 = q^2$
(c.) $P^2 = q^2 + 1$
(d.) None of these

Q.18 If $A = \begin{pmatrix} i & -i \\ -i & i \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ then A^8 equals

(a.) 128 B
(b.) 32 B
(c.) 16 B
(d.) 64 B

Sol. A = iB $\Rightarrow A^2 = i^2B^2 = (-1)\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = -2B$

$\Rightarrow A^4 = (-2B)^2 = 4B^2 = 4(2B) = 8B \qquad \Rightarrow A^8 = (8B)^2 = 64B^2 = 64(2B) = 128B$

Q.19 The values of k for which the system of equations $x + ky - 3z = 0,\ 3x + ky - 2z = 0,$ $2x + 3y - 4z = 0$ has a non –trivial solution is (are)

(a.) $\frac{21}{10}$
(b.) $\frac{31}{10}$
(c.) -5
(d.) 4

Sol. From (1) and (2), $2x+z=0 \Rightarrow 2x=-z$. From (3), we get $3y=5z$ $\therefore$ $x = -\frac{1}{2}z, y = \frac{5}{3}z$

Substituting this in (1) we get $-\frac{1}{2}z + \frac{5k}{3}z - 3z = 0. \Rightarrow \left(\frac{5k}{3} - \frac{7}{2}\right)z = 0$ Now, $z \neq 0 \Rightarrow k = \frac{21}{10}$.

Q.20 If $\theta, \phi \in R$, then the determinant $\Delta = \begin{vmatrix} \cos\theta & -\sin\theta & 1 \\ \sin\theta & \cos\theta & 1 \\ \cos(\theta+\phi) & -\sin(\theta+\phi) & 0 \end{vmatrix}$ lies in the interval

(a.) $\left[-\sqrt{2}, \sqrt{2}\right]$ (b.) $\left[-1, 1\right]$ (c.) $\left[-\sqrt{2}, 1\right]$ (d.) $\left[-1, \sqrt{2}\right]$

Sol. Applying $R_3 \to R_3 - \cos\varphi R_1 \sin\varphi R_2$, we get

$$\Delta = \begin{vmatrix} \cos\theta & -\sin\theta & 1 \\ \sin\theta & \cos\theta & 1 \\ 0 & 0 & \sin\varphi - \cos\varphi \end{vmatrix} = (\sin\varphi - \cos\varphi)(\cos^2\theta + \sin^2\theta)$$

$= \sqrt{2}\left\{\frac{1}{\sqrt{2}}\sin\varphi = \frac{1}{\sqrt{2}}\cos\varphi\right\} = \sqrt{2}\sin(\varphi - \pi/4)$ As $-1 \le \sin(\varphi - \pi/4) \le 1, -\sqrt{2} \le \sqrt{2}\sin(\varphi - \pi/4) \le \sqrt{2}$
or $-\sqrt{2} \le \Delta \le \sqrt{2}$.

Q.21 Given 2x - y + 2z =2 , $x - 2y + z = -4$, $x + y + \lambda z = 4$, then the values of λ such that the given system equations has no solution, is

(a.) 3 (b.) 1 (c.) 0 (d.) -3

Sol. Form the first two equation, we get x+y=6-z
Putting the value in the last equation we get
6+(-1+λ)z=4=(λ-1)z=-2. For the system of equations to have no solution λ=1

Q.22 If a +b + c=0, then a root of the equation $\Delta = \begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$ is

(a.) 1 (b.) -1 (c.) $a^2 + b^2 + c^2$ (d.) 0

Sol. Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\Delta = \begin{vmatrix} a+b+c-x & c & b \\ a+b+c-x & b-x & a \\ a+b+c-x & a & c-x \end{vmatrix} = \begin{vmatrix} -x & c & b \\ -x & b-x & a \\ -x & a & c-x \end{vmatrix}$$

[$\because$ a+b+c=0] Δ clearly equals 0 when x = 0

Q.23 The values of λ for which the system of equations
x+y-3=0, $(1+\lambda)x + (2+\lambda) - 8 = 0$, $x - (1+\lambda)y + (2+\lambda) = 0$ has a non-trivial solutions,

(a.) -5/3, 1 (b.) 2/3,-3 (c.) -1/3,-3 (d.) 0

Sol. The given system of equations has a non trivial solution if

$$\Delta = \begin{vmatrix} 1 & 1 & -3 \\ 1+\lambda & 2+\lambda & -8 \\ 1 & -(1+\lambda) & 2+\lambda \end{vmatrix} = 0$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 + 3C_1$, we ge $\Delta = \begin{vmatrix} 1 & 0 & -3 \\ 1+\lambda & 1 & -5+3\lambda \\ 1 & -2-\lambda & 5+\lambda \end{vmatrix} = 0$

$\Rightarrow (5+\lambda) + (2+\lambda)(3\lambda - 5) = 0 \Rightarrow 5 + \lambda + 6\lambda - 10 + 3\lambda^2 - 5\lambda = 0 \Rightarrow \lambda = -5/3$ or $\lambda = 1$.

Q.24 If the system of equations x-ky-z=0, kx−y-z=0,x+y-z=0 has a non-zero solution, then the possible values of k are

(a.) -1, 2 (b.) 1,2 (c.) 0, 1 (d.) -1,1

Sol. As the given system has a non-zero solution,

$$0=\begin{vmatrix}1 & -k & -1\\ k & -1 & -1\\ 1 & 1 & -1\end{vmatrix}=\begin{vmatrix}1+k & -k-1 & -1\\ 1+k & -2 & -1\\ 0 & 0 & -1\end{vmatrix} \quad \text{using } C_1\to C_1-C_2, C_2\to C_2+C_3]$$

$\Rightarrow$ 0=(-1)[(1+k)(-2)-(1+k)(-k-1)] $\Rightarrow$ 0=(1+k)(-2+k+1) $\Rightarrow$ k=-1, 1

Q.25 The number of real values of a for which the system of equations x+ay-z=0,2x-y+az=0,ax+y+2z=0 has a non-trivial solution, is

(a.) 3 (b.) 1 (c.) 0 (d.) infinite

Q.26 If $\Delta=\begin{vmatrix}-a & 2b & 0\\ 0 & -a & 2b\\ 2b & 0 & -a\end{vmatrix}=0$, then

(a.) 1/b is a cube root of unity
(b.) a is one of the cube roots of unity
(c.) b is one of the cube roots of 8
(d.) a/b is a cube roots of 8

Sol. Show $\Delta=-a^3+8b^3$ Thus, $\Delta=0\Rightarrow\left(\frac{a}{b}\right)^3=8$ $\Rightarrow$ a/b is a cube root of 8.

Q.27 If a, b, c are non-zero real numbers, the $\Delta=\begin{vmatrix}1 & ab & \frac{1}{a}+\frac{1}{b}\\ 1 & bc & \frac{1}{b}+\frac{1}{c}\\ 1 & ca & \frac{1}{c}+\frac{1}{a}\end{vmatrix}$ equals

(a.) 0
(b.) bc+ca+ab
(c.) $a^{-1}+b^{-1}+c^{-1}$
(d.) abc-1

Sol. Write $R_1\to R_2-R_1$ and $R_3\to R_3-R_1$ $\Delta=abc\begin{vmatrix}1 & \frac{1}{c} & \frac{1}{a}+\frac{1}{b}\\ 1 & \frac{1}{a} & \frac{1}{b}+\frac{1}{c}\\ 1 & \frac{1}{b} & \frac{1}{c}+\frac{1}{a}\end{vmatrix}$ and use $C_2\to C_2+C_1$, to show that Δ = 0

Q.28 If $\Delta=\begin{vmatrix}1+Y & 1-Y & 1-Y\\ 1-Y & 1+Y & 1-Y\\ 1-Y & 1-Y & 1+Y\end{vmatrix}$ =0,then values of y are

(a.) 0,3 (b.) 2,1 (c.) -1,3 (d.) 0,2

Sol. Using $C_1\to C_1+C_2+C_3$, we get

$$\Delta = (3-y)\begin{vmatrix} 1 & 1-y & 1-y \\ 1 & 1+y & 1-y \\ 1 & 1-y & 1+y \end{vmatrix} \text{ Applying } R_3 \to R_3 - R_2,\ R_2 \to R_2 - R_1\text{, we get}$$

$$\Delta = (3-y)\begin{vmatrix} 1 & 1-y & 1-y \\ 0 & 2y & 0 \\ 0 & -2y & 2y \end{vmatrix} = (3-y)(4y^2) \quad \therefore \quad \Delta = 0 \Rightarrow y = 0 \text{ or } y = 3.$$

Q.29 If A is symmetric matrix and B is a skew symmetric matrix, then for any $n \in N$, which of the following is not correct?

(a.) A^n is symmetric

(b.) A^n is symmetric, if n is odd.

(c.) B^n is skew symmetric, if n is odd

(d.) B^n is symmetric, if n is even.

Sol. If A is symmetric, then every integral power of A is symmetric.

If B is skew symmetric then every odd integral powers of B are skew symmetric and every even integral powers of B are symmetric.

Q.30 If $A = \begin{pmatrix} \frac{-1+i\sqrt{3}}{2i} & \frac{-1-i\sqrt{3}}{2i} \\ \frac{1+i\sqrt{3}}{2i} & \frac{1-i\sqrt{3}}{2i} \end{pmatrix}$, $i = \sqrt{-1}$ and $f(x) = x^2 + 2$, then f(A) is equal to:

(a.) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(b.) $\left(\frac{3-i\sqrt{3}}{2}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(c.) $\left(\frac{5-i\sqrt{3}}{2}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(d.) $\left(2+i\sqrt{3}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Sol. $\because\ \omega = \frac{-1+i\sqrt{3}}{2}$ and $\omega^2 = \frac{-1-i\sqrt{3}}{2}$

Also, $\omega^3 = 1$ and $\omega + \omega^2 = -1$

$$\text{Then, } A = \begin{bmatrix} \frac{\omega}{i} & \frac{\omega^2}{i} \\ -\frac{\omega^2}{i} & -\frac{\omega}{i} \end{bmatrix} = \frac{\omega}{i}\begin{bmatrix} 1 & \omega \\ -\omega & -1 \end{bmatrix} \therefore \quad A^2 = \frac{\omega^2}{i^2}\begin{bmatrix} 1 & \omega \\ -\omega & -1 \end{bmatrix}\begin{bmatrix} 1 & \omega \\ -\omega & -1 \end{bmatrix} = -\omega^2\begin{bmatrix} 1-\omega^2 & 0 \\ 0 & 1-\omega^2 \end{bmatrix}$$

$$= \begin{bmatrix} -\omega^2 + \omega^4 & 0 \\ 0 & \omega^2 + \omega^4 \end{bmatrix} = \begin{bmatrix} -\omega^2 + \omega^2 & 0 \\ 0 & -\omega^2 + \omega \end{bmatrix}$$

$$\because f(x) = x^2 + 2 \quad \therefore f(A) = A^2 + 2I = \begin{bmatrix} -\omega^2 + \omega & 0 \\ 0 & -\omega^2 + \omega \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -\omega^2 + \omega + 2 & 0 \\ 0 & -\omega^2 + \omega + 2 \end{bmatrix}$$

$$= \left(-\omega^2 + \omega + 2\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (3 + 2\omega)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \left(3 + 2\left(\frac{-1+i\sqrt{3}}{2}\right)\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \left(2 + i\sqrt{3}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Q.31 If α,β,γ are the roots of $x^3+px^2+q=0$, where $q\neq 0$, and $\Delta=\begin{vmatrix}1/\alpha & 1/\beta & 1/\gamma\\ 1/\beta & 1/\gamma & 1/\alpha\\ 1/\gamma & 1/\alpha & 1/\beta\end{vmatrix}$ then Δ equals

(a.) -p/q (b.) 1/q (c.) p^2q (d.) 0

Sol. We have $\beta\gamma+\gamma\alpha+\alpha\beta=0$. We can write Δ as

$$\Delta=\frac{1}{\alpha^3\beta^3\gamma^3}\begin{vmatrix}\beta\gamma & \gamma\alpha & \alpha\beta\\ \gamma\alpha & \alpha\beta & \beta\gamma\\ \alpha\beta & \beta\gamma & \gamma\alpha\end{vmatrix}=\frac{1}{\alpha^3\beta^3\gamma^3}\begin{vmatrix}\beta\gamma+\gamma\alpha+\alpha\beta & \gamma\alpha & \alpha\beta\\ \gamma\alpha+\alpha\beta+\beta\gamma & \alpha\beta & \beta\gamma\\ \alpha\beta+\beta\gamma+\gamma\alpha & \beta\gamma & \gamma\alpha\end{vmatrix}$$ [using [$C_1\to C_1+C_2+C_3$]

$$=\frac{1}{\alpha^3\beta^3\gamma^3}\begin{vmatrix}0 & \gamma\alpha & \alpha\beta\\ 0 & \alpha\beta & \beta\gamma\\ 0 & \beta\gamma & \gamma\alpha\end{vmatrix}=0$$ [all zero property

Q.32 If $\Delta_1=\begin{vmatrix}b+c & a-b & a\\ c+a & b-c & b\\ a+b & c-a & c\end{vmatrix}$ and $\Delta_2=\begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix}$, then $\Delta_1-\Delta_2$ equal

(a.) 0 (b.) 3abc (c.) 6abc (d.) $2(a^3+b^3+c^3)$

Sol. Using $C_1\to C_1+C_3$ and $C_2\to C_2-C_3$ in Δ_1, we get

$$\Delta_1=\begin{vmatrix}a+b+c & -b & a\\ a+b+c & -c & b\\ a+b+c & -a & c\end{vmatrix}$$ Using $C_1\to C_1+C_2-C_3$, we get

$$\Delta_1=\begin{vmatrix}c & -b & a\\ a & -c & b\\ b & -a & c\end{vmatrix}=\begin{vmatrix}a & -b & c\\ b & -c & a\\ c & -a & b\end{vmatrix}=\begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix}\Rightarrow\Delta_1=\Delta_2\Rightarrow\Delta_1-\Delta_2=0.$$

Q.33 If α,β,γ are real numbers, then the determinant $\Delta=\begin{vmatrix}\sin^2\alpha & \cos 2\alpha & \cos^2\alpha\\ \sin^2\beta & \cos 2\beta & \cos^2\beta\\ \sin^2\gamma & \cos 2\gamma & \cos^2\gamma\end{vmatrix}$ equals

(a.) 0 (b.) -1

(c.) $\sin^2\alpha+\sin^2\beta+\sin^2\gamma$ (d.) None of these

Sol. Use $C_2\to C_2-(C_3-C_1)$ to show that C_2 consists of all zero.

Q.34 If $x\in R$, the determinant

$$\Delta=\begin{vmatrix}1 & \cos x & 0\\ -1 & 1-\cos x & \sin x+\cos x\\ 0 & -1 & 1-\sqrt{2}\sin(x+\pi/4\end{vmatrix}$$ equals

(a.) 0 (b.) -1 (c.) 1 (d.) None of these

Sol. Using $R_2\to R_2+R_1$, we get

$$\Delta=\begin{vmatrix}1 & \cos x & 0\\ 0 & 1 & \sin + \cos x\\ 0 & -1 & 1-(\sin x+\cos x)\end{vmatrix}$$ Using $R_3\to R_3+R_2$, we get $\Delta=\begin{vmatrix}1 & \cos x & 0\\ 0 & 1 & \sin + \cos x\\ 0 & 0 & 1\end{vmatrix}=1$

Q.35 If $\theta \varepsilon R$, maximum value of $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$ is

(a.) 1/2 (b.) $\sqrt{3}/2$ (c.) $\sqrt{2}$ (d.) $3\sqrt{2}/4$

Sol. Use $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$ to obtain $\Delta = \sin\theta\cos\theta = \frac{1}{2}\sin(2\theta)$

Q.36 If $\Delta = \begin{vmatrix} 1 & x & x^2 \\ x & x^2 & 1 \\ x^2 & 1 & x \end{vmatrix}$ = - 7 and $\Delta_1 = \begin{vmatrix} x^3-1 & 0 & x-x^4 \\ 0 & x-x^4 & x^3-1 \\ x-x^4 & x^3-1 & 0 \end{vmatrix}$, then

(a.) $\Delta = 7$ (b.) $\Delta = 343$ (c.) $\Delta = -49$ (d.) $\Delta = 49$

Sol. Note that

$\Delta_1 = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}$ where Cij cofactor of (i, j)th element of

$\begin{vmatrix} 1 & x & x^2 \\ x & x^2 & 1 \\ x^2 & 1 & x \end{vmatrix}$ $\therefore$ $\Delta_1 = \Delta^2 = 49$

Q.37 Suppose A is square matrix such that $A^3 = I$, then $(A+I)^3 + (A-I)^3 - 6A$ equals

(a.) I (b.) 2 I (c.) A (d.) 3 A

Q.38 If $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ {}^mC_1 & {}^{m+3}C_1 & {}^{m+6}C_1 \\ {}^mC_2 & {}^{m+3}C_2 & {}^{m+6}C_2 \end{vmatrix} = 2^\alpha\, 3^\beta\, 5^\gamma$, then $\alpha+\beta+\gamma$ is equal

(a.) 3 (b.) 5 (c.) 7 (d.) None of these

Sol. We have $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ m & m+3 & m+6 \\ \frac{1}{2}m(m-1) & \frac{1}{2}(m+3)(m+2) & \frac{1}{2}(m+6)(m+5) \end{vmatrix}$

Applying $C_3 \to C_3 - C_2, C_2 \to C_2 - C_1$, we get

$\Delta = \begin{vmatrix} 1 & 0 & 0 \\ m & 3 & 3 \\ \frac{1}{2}m(m-1) & 3(m+1 & 3(m+4) \end{vmatrix} = 3^2(m+4-m-1) = 3^3 \quad \therefore \quad \alpha+\beta+\gamma = 3$

Q.39

The determinant $\begin{vmatrix} a & a+d & a+2d \\ a^2 & (a+d)^2 & (a+2d)^2 \\ 2a+3d & 2(a+d) & 2a+d \end{vmatrix} = 0$. then :

(a.) d = 0 (b.) a + d = 0 (c.) d = 0 or a + d = 0 (d.) none of these

Sol Use $C_3 \rightarrow C_3 - C_2$, $C_2 \rightarrow C_2 - C_1$.

Q.40

If there are two values of 'a' for which $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix}$ is 86, then the sum of these values

(a.) 4 (b.) 5 (c.) -4 (d.) -5